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AN INTRODUCTION TO REACTANCE

Take a step beyond resistive-circuit analysis.

Simple resistive circuits are fairly easy understand, but circuits with capacitors and inductors, known collectively as reactive components, are more elusive. This article will attempt to shed light on reactive circuits by comparing them to resistive ones.

Let's start by examining the simple resistive circuit shown in Fig. 1A. When DC is wupplied by the battery, the current developed can be determined by using Ohm's Law, the battery voltage (E), and the resistors value (R). When an AC source is used (Fig. 1B), the instantaneous value of current flowing through the resistor varies with the alternating output voltage of the generator. Ohm's Law applies to the circuit just as it does to the DC circuit; peak current can be calculated from the generator's peak voltage, and rms current from the rms voltage. The graph of one cycle of AC (Fig. 1C) shows that when the voltage reaches a peak, the current also peaks; when the voltage crosses the zero line, so does the current. The two waveforms are said to be in phase, and this condition occurs in any circuit that contains pure resistance with no reactive components.

Inductance. The first reactive element we shall examine is inductance. The basic laws of electromagnetism tell us that whenever current flows through a conductor, a magnetic field is developed around that conductor. Similarly, whenever a conductor posses through a magnetic field, or a magnetic field cuts through a conductor, a current is induced in that conductor.

Figure 2A shows a coil wired into a simple DC circuit. When the switch is operated to connect the battery, current starts to flow through the windings of the coil. As the current builds up, a magnetic field expands from each turn of the coil and cuts across neighboring turns. That expanding field induces a separate current in the coil in the opposite direction to the energizing current from the battery. The induced current, known as counter-EMF

(or electromotive force), can never be as great as the original current, but since it is of opposite polarity, it restricts the rate at which current in the coil can increase. When the field around each turn of wire stops expanding, there is no longer any induced counter-EMF, so the final value of current is limited only by the resistance of the circuit (just R if we disregard the tiny resistance in the coil's windings) It is important to realize that the time constant is affected not only by the inductor itself, but also by the resistance of the circuit to which it is connected.

The build-up of current follows an exponential curve, as shown in Fig. 2B. The period between the application of power and the current reaching 63.2% of its maximum value is called the circuit's time constant. The time constant's value in seconds can be calculated by dividing the inductance in henries by the circuit resistance, in ohms. For most practical purposes, the current is considered to have reached its maximum value after a period equal to five times the constant.

Several factors affect the inductance of a coil: It can be increased by adding more turns, using an iron core, reducing the spacing between adjacent turns, and altering the coil's overall diameter. For example, if a 10-henry coil is used with a circuit resistance of 50 ohms, the time constant is 0.2 second. Current through the coil would reach 63.2% of its final value after 0.2 second, and would be considered to have reached maximum (over 99%) after approximately 1 second. If the coil was changed to one of 20 henrys by adding more turns, the time constant would be increased to 0.4 second. Current would now reach its maximum value 2 seconds after power was applied.

Assume now that the switch in Fig. 2 is moved back to its original position, connecting the coil to the resistor. Even though the battery is disconnected, current will still flow for a short time. With the energizing current removed, the magnetic field around each loop of the coil starts to collapse, and in doing so generates EMF within the inductor. It is that induced EMF that attempts to maintain current flow. After one time constant, the current will have dropped to 36.8% of its maximum value (i.e., 63.2% toward zero), and is considered to be zero after five time constants.

The result of connecting an inductive circuit to an AC voltage source (as in Fig. 3A) is shown in Fig. 3B. The voltage starts to rise from zero at the beginning of a cycle, but as you might guess, the circuit current does not rise instantaneously as it would in a purely resistive circuit. The voltage will have passed its positive peak and dropped toward zero before the inductance allows the current to reach its peak value. In other words, the instantaneous current flowing in the circuit is less than what would be calculated from the resistance of the coil wire alone.

The opposition to current flow due to the inductor's induced EMF is called inductive reactance, denoted X_L , and is measured in ohms. If the frequency of the applied AC is reduced, the coil current will reach a higher value before the voltage waveform passes its peak and drops toward zero, so inductive reactance is directly proportional to the applied frequency as follows:

$$X_L = 2\pi fL$$

The formula assumes that the value of inductance (L) is given in henrys and the frequency (f) in cycles-per-second, or hertz.

In a theoretical circuit consisting of pure inductance, with no resistance, the current lags behind the voltage by exactly 90 degrees: When voltage is maximum, current is minimum,

and vice versa. Since there is no resistance, the current in the circuit can be determined by substituting X_L for R in the Ohm's Law equation.

Capacitance. The electrically speaking, the opposite of inductance is capacitance. Lets discuss how current flows through a capacitor as it charges from a DC source (shown In Fig. 4A). When the switch is closed, the current immediately rises to a value set by the limiting resistor (R). Electrons from the negative pole of the battery flow to one plate of the capacitor to give it a negative charge, while at the same time, electrons are drawn from the other plate to the positive battery pole. The current gradually drops as the plates charge, and in an ideal capacitor the current would be zero when the plates were charged to capacity.

At the instant that current is applied, there is no voltage across the capacitor (see Fig. 4B). As charge accumulates on the plates and the charging current drops, the capacitors voltage rises. The plotted curve of the raftage rise is exponential, and is identical in shape to the curve shown earlier for the rise of current in an inductor.

The capacitive time constant is defined as the period between the application of charging current and the point at which the capacitor voltage reaches 63.2% of the supply voltage. It is equal to the product of the capacitance and overall circuit resistance. With the capacitance specified in farads and resistance in ohms, the result will be in seconds. Once again, the circuit is considered to have reached its final values, with the capacitor fully charged, after approximately five time constants.

When the circuit is switched to discharge the capacitor, both the current and voltage decrease gradually, as would be expected. After a period equal to one time constant, the capacitor voltage will be 36.8% of its full value, and after five such periods the capacitor is considered to be fully discharged.

Figure 5A shows the capacitor in an AC circuit. Notice that the relationship between current and voltage in a purely capacitive circuit, as reflected in the graph in Fig. 5B, is exactly opposite to that in an inductive circuit--the current leads the raftage by 90 degrees. Whereas the inductor opposes changes in circuit current by generating a counter EMF, the capacitor opposes changes in voltage by storing and releasing electrons from its plates. However, the overall affect on current flow is similar: the effect is greater than can be accounted for by resistance alone.

The opposition to current flow in a capacitor is called capacitive reactance, X_c , and is also measured in ohms. As the frequency of the applied AC is reduced, the capacitor will have longer to charge before the supply polarity changes, so the overall current will be lower. Capacitive reactance is therefore inversely proportional to frequency, as indicated by:

$$X_c = 1/(2 \pi f C)$$

where the frequency (f) is in hertz, and the capacitance (C) is in farads. Note that is the exact opposite of inductive reactance.

Phase Angle and Vectors. The effects of pure capacitance or inductance can be understood quite simply by remembering that they cause current and voltage to be 90 degrees out-of-phase. However, the situation becomes more complex when both reactive components and resistance are combined, which is the case in any practical circuit.

Look at the circuit of Fig. 6 as we proceed. The components in the circuit shown are in series, so the current flow must be the same through path of them. The resistor has a value of 50 ohms, and we have assumed that the inductor has an inductive reactance of 25 ohms at the frequency in use. The current flow is 1.79 amps, and the voltage drops measured across the resistor and inductor are 89.5V and 44.75V respectively. At first sight, the voltages shown may appear to be incorrect, because together they total more than the supply voltage of 100 volts. That apparent discrepancy, however, is caused by the fact that the voltage across the resistor is out of phase with that across the inductor.

The voltage and current in the resistive portion of the circuit are in phase with each other. In the inductive portion of the circuit, the voltage leads the current by 90 degrees. Since the current at all points in a series circuit must be of the same phase, it follows that E_R and E_L are out of phase.

One way to represent current and voltage in circuits like that is to use vectors—arrows that start at the origin of a coordinate system and point in a particular direction (see Fig. 7A). The size of an arrow indicates the magnitude of whatever the arrow represents—the bigger the arrow, the larger the value. The angle an arrow makes with the X-axis indicates its phase in degrees. The positive X-axis will be zero degrees, and the degrees increase as you move in a counterclockwise direction.

Because the current in the series circuit of Fig. 6 flows through both components with the same phase, we will use the current vector as the zero-degree reference. In other words, the current vector will lie on the X-axis. Voltage E_R is in phase with the current, so its vector will be placed on that axis, too.

The voltage across the coil, E_L , is 90 degrees ahead of the current, so its vector points straight up (90 degrees) from the X-axis. Because each vector is drawn to scale, we can draw the source-voltage vector (E) by starting it at the origin and ending it at the maximum values of E_L and E_R as shown. The angle of vector E (which is θ) is the phase between the current and source voltage, and its length indicates the voltage's magnitude.

The basic vector diagram can be rearranged to form a right triangle, with the hypotenuse representing the longest vector (see Fig. 7B). Pythagoras' Theorem can be used to calculate any vector when the other two are known, as follows:

$$E = \sqrt{E_R^2 + E_L^2}$$

so scale drawings are not needed to determine magnitudes.

Vector representation also allows us to use trigonometric functions to determine voltages when we know only one voltage and a phase angle, or to determine a phase angle when we know only two voltages. For our example these relationships are:

$$\begin{aligned}\sin \theta &= E_L/E \\ \cos \theta &= E_R/E \\ \tan \theta &= E_L/E_R\end{aligned}$$

From any of these relations, we can determine that the phase difference between the current and the supply voltage in our example is 26.56 degrees.

A little experimentation with different values for E_L and E_R will reveal some useful tips to remember. Whenever a circuit is purely resistive, the phase angle is zero, because current and voltage are in phase. As the inductive reactance increases, the phase angle becomes greater, until it reaches 45 degrees when resistance and reactance are equal in value. By the time a circuit contains pure reactance, with no resistance, the phase angle has increased to 90 degrees.

Since the current flow is the same through all components in a series circuit, it follows that the voltage drop across any component in that circuit is proportional to the resistance or inductive reactance of that component. In other words, if we draw vectors for the resistance and reactance of the circuit, they would be proportional to the ones we've drawn for the voltages (see Fig. 7C). The resistance, R , would be at 0 degrees and inductive the reactance, X_L , at 90 degrees.

The combined effect of resistance and reactance is called impedance, which is represented by the symbol Z . It is that value that must be used to calculate the current in a reactive circuit when the supply voltage is known; just dividing the source voltage by the resistance plus the reactance will yield an incorrect answer, since the voltages involved are not in phase.

The series RC circuit in Fig. 8A is described by the vector diagrams in Figs. 8B and 8C. Once again, the current is used as the zero-degree reference point, with E_R at 0 degrees since it is in phase with the current. Recall that in a capacitor, the voltage lags 90 degrees behind the current, so its voltage vector (E_C) is drawn downward. The phase angle of such a circuit is sometimes given a negative value, although it is just as acceptable to specify "leading" or "lagging" instead. All the same trigonometric equations and the Pythagorean Theorem can be applied to the vectors.

Parallel-Circuit Vectors. Parallel circuits containing capacitors and inductors can also be analyzed with vector diagrams. However, the vectors used are current vectors since the voltages across each component must be equal and in phase with each other.

Figure 9A shows a parallel RL circuit and its resulting vector diagrams. Once again the vector for the resistor is placed at zero degrees. It is now a simple matter to plot the vector for the coil's current, I_L . The current in an inductor lags the voltage by 90 degrees, so the vector is drawn downward.

Figure 9B shows the vector diagram for a parallel RC circuit. Notice that the capacitive and inductive vectors for parallel circuits are drawn the opposite way to those for series circuits. That is simply because we are examining current, not voltage.

Just as the resistance of a parallel resistor network is always less than the value of the smallest resistor, so the impedance of a parallel RC or RL circuit is smaller than both the individual resistance and reactance. Vectors can be used to determine parallel circuit impedance, but the reciprocal values of R , X , and Z must be used. That makes impedance calculations a little more complex for parallel circuits. All the trigonometric functions previously mentioned are just as applicable to the parallel circuit, so long as care is taken to get the vectors drawn correctly in the first place. Mathematically, for Fig. 9A:

$$I = \text{square root } (I_{R\{\text{sup } 2\}} + I_{L\{\text{sup } 2\}}) \quad 1/Z = \text{square root } ((1/R)^2 + (1/X_L)^2)$$

and for Fig. 9B

$$I = \text{square root } (I_{R(\text{sup } 2)} + I_{C(\text{sup } 2)} 1/Z) = \text{square root } ((1/R)^2 + (1/X_C)^2)$$

Power. The power consumption of a purely resistive AC circuit is easy to determine: Simply calculate the product of the rms current and rms voltage to obtain average power. Figure 11A shows a graphical way in which instantaneous power consumption can be calculated by plotting current and voltage on the same axes and then performing successive multiplications to plot the power curve.

The same principle can be applied to the reactive circuit (see Fig. 11B). Recall that in a circuit containing just pure inductance, current lags voltage by 90 degrees. Plotting the power curve for this circuit yields an alternating waveform that is centered on zero. In other words, the net power consumption of an inductive circuit is nil.

That may seem a little confusing, so lets discuss why. During the positive portion of the waveform, the inductor takes energy and stores it in the form of a magnetic field. During the negative portions of the curve, the field collapses and the coil returns energy to the circuit. A similar situation occurs with pure capacitance, except that the capacitor stores energy as an electrostatic field and the current/voltage phase relationship is reversed.

As resistance is introduced to a circuit, the phase angle becomes less than 90 degrees and the power curve will shift to a more positive value, showing that the circuit is faking more energy than if is returning. However, the capacitive or inductive port of the circuit still stores and releases energy and consumes no power what-so-ever--the power loss is due entirely to the resistance. Remember this basic rule: Only the resistive part of a circuit consumes power.

Take the circuit in Figure 11A as an example, Using the Pythagorean the orem, the resistance of 100 ohms and capacitive reactance of 50 ohms gives a combined impedance (Z) of approximately 112 ohms as follows:

$$\begin{aligned} Z &= \text{square root } (100^2 + 50^2) \\ &= \text{square root } (10,000 + 2500) \\ &= \text{square root } (12,500) \\ &= 111.8 \end{aligned}$$

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By Ohm's law, that would allow a current of about 1 amp to flow in the circuit. The voltage drop across the resistor will be 100 volts, so the "true power" dissipated by that resistor will be 100 watts.

We know that the capacitive part of the circuit consumes no power, and yet multiplying the source voltage by circuit current yields on answer of 112 watts. The difference is accounted for, once again, by the difference in phase between the various voltages. As opposed to true power, the figure obtained by multiplying the source voltage and current is known as "apparent power," and it is specified as 112VA or 112 volt-amperes. The true power of the circuit can never be greater than the apparent power, and the ratio of true power in wafts to apparent power in volt-amps is called the power factor.

A vector diagram can again be used to analyze the power factor. If the phase angle of the circuit is known. the power factor can be calculated directly by taking the cosine of that angle. In the circuit shown, that figure is the same as the ratio of  $E_R$  to source voltage. There are always several ways to obtain the power factor, depending upon the available

data. In a purely resistive circuit, the true power is the same as the apparent power, so the power factor is 1. In a purely inductive or capacitive circuit the true power is 0, no matter what the apparent power, so the power factor is 0. Power factor is a very important consideration in heavy industrial power distribution, since cables must be able to handle the apparent-power load.

DIAGRAM: Fig. 1. DC resistive circuits (A) and AC resistive circuits (B) both obey a simple version of Ohm's Law. So in a resistive AC circuit, current and voltage vary in step with one another (C).

DIAGRAM: Fig. 2. The inductor in the circuit shown in A causes current to lag behind the applied voltage, as shown in B.

DIAGRAM: Fig. 3. In a purely inductive AC circuit (A), the current waveform lags behind the voltage waveform by 90 degrees (B).

DIAGRAM: Fig. 4. The charging action of the capacitor in A causes voltage to lag behind current, as shown in B.

DIAGRAM: Fig. 5. In a purely capacitive AC circuit (A), the current waveform leads the voltage waveform by 90 degrees (B).

DIAGRAM: Fig. 6. Voltages in RL circuits like this one, or in RC circuits, are not in phase and cannot be added directly.

DIAGRAM: Fig. 7. Vectors can be used to show the relationship between voltages in a reactive circuit (A). By rearranging vectors (B), one can see their geometric relationship. Vectors can also be used to describe circuit impedance (C), too.

DIAGRAM: Fig. 8. A capacitive AC circuit (A) falls prey to vector analysis for its voltages (B) and impedance (C) just like an inductive circuit.

DIAGRAM: Fig. 9. Vectors can be used to analyze parallel inductive (A) and capacitive (B) circuits, but current flow, not voltage, is the property of interest. Also, the only inverse of the impedance vectors are examined.

DIAGRAM: Fig. 10. While power dissipation in a resistive circuit has a non-zero average value (A), in a reactive circuit there is no average or net power loss (B).

DIAGRAM: Fig. 11. In a reactive circuit, the true power dissipated in its resistance and the reactive power supplied to its reactance vectorially sum to produce an apparent power vector, even though reactive energy is returned to the circuit.

ILLUSTRATION

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