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measurement

The branch of arithmetic that is concerned with measurement of length, surface, and volume is called mensuration. Mensuration deals with so-called geometrical figures, such as squares, triangles, circles, cubes, spheres, and cylinders. Measurements of these figures are much used in surveying land, in building, plastering, and papering, in drawing and engineering, and in finding the capacity of boxes, tanks, and ships. (See also [Geometry](#).)

All measurements of surface and volume are based upon measurements of length. The chief traditional units of length in English-speaking countries are the inch, foot, yard, rod, and mile. These have largely been replaced by such metric units of length as millimeter, centimeter, meter, and kilometer. (For the origin of these units and for tables of English and metric measures, see [Weights and Measures](#).)

The Perimeter of a Plane Figure

Linear measure is the measurement of a line. A line has only one dimension-length. A plane, or flat, surface has two dimensions-length and width. The perimeter of a plane figure is the distance around it.

Figure 1 shows a scale drawing of a garden plot 50 yards long and 10 yards wide. Each half inch represents 10 yards. The garden plot is in the form of a rectangle. A rectangle is a figure with four straight sides and four square corners.

Suppose we want to know how many yards of fence will be required to enclose this rectangle. The perimeter is seen to be 2 times the length plus 2 times the width. Calling the perimeter P , the length l , and the width w , we can write the formula for the perimeter of any rectangle:

The number of yards of fence required is $250 + 210 = 100 + 20 = 120$, or $2(50 + 10) = 260 = 120$.

Measurement of Surface, or Area

We have seen that we measure length by finding out how many linear units of a certain standard size are contained in the line we are measuring. We measure area by finding out how many square units of a certain size are contained in the surface we are measuring. The square units commonly used are the square inch, the square foot, the square yard, the square rod, the acre, and the square mile.

The Rectangle.

Figure 2 shows small scale drawings of a rectangle that measures 3 inches by 4 inches. Each 1/4 inch in the scale drawings represents 1 inch. The small square in the corner of Rectangle A represents a square 1 inch long and 1 inch wide (a square inch).

Rectangle B is divided into 3 vertical strips. One of the 3 strips is divided into 1-inch squares. We see that there are 4 square inches in 1 strip. Then in 3 strips there are 3 times 4 square inches, or 12 square inches.

Rectangle C is divided into 4 horizontal strips. One of the strips is divided into 3 square inches. Then in the 4 strips, there are 4 times 3 square inches, or 12 square inches. We see that the area of a rectangle in square inches is equal to the length in inches times the width in inches. If the dimensions were given in feet, the area would be in square feet; if in yards, the area would be in square yards; and so on.

If A stands for area, l for length, and w for width, the formula for the area of a rectangle is:

If the sides of a rectangle are all equal, the rectangle is a square. If we call one side of a square s, the area of a square is given by the formula:

We read s^2 as 's squared' (see [Arithmetic](#)).

Using this formula, we can work out the units of square measure. As Figure 3 shows, if a square is 1 foot long and 1 foot wide, its area is 1 square foot. Since there are 12 inches in a foot, the area of a square foot is 1212, or 144 square inches. Thus there are 144 square inches in one square foot.

If a square is 3 feet long and 3 feet wide, the area is 9 square feet, or 1 square yard.

A rod is 1612 feet. An acre contains 160 square rods. The area of a field 80 rods long and 40 rods wide is 8040, or 3,200 square rods. Dividing 3,200 by 160 we find the number of acres to be 20.

The Parallelogram.

Parallel lines may be defined as lines that run in the same direction and never meet. A parallelogram is a four-sided plane of which the opposite sides are parallel. All rectangles are parallelograms but not all parallelograms are rectangles.

Either pair of opposite sides may be called the bases. The altitude is measured by the perpendicular between the bases. The perpendicular is a line that forms square corners with the base line. In a rectangle the altitude, or perpendicular, is a side of the figure.

Figure 4A is a parallelogram. If we cut along the dotted line, turn over the cut-off piece, and move it to the other side, we have a rectangle (Figure 4B). The area of this rectangle is 25, or 10 square inches. The formula for the area of a parallelogram is therefore:

The Trapezoid.

A trapezoid has only two parallel sides-the bases. Figure 5A has a lower base (B) of 10 feet and an upper base (b) of 6 feet. The altitude (a) is 4 feet. If two such trapezoids are placed end to end, as in Figure 5B, they form a parallelogram. Each base is equal to $B+b$, or 10 feet+6 feet, or 16 feet. The area is 416, or 64 square feet. The area of the trapezoid is one half the area of the parallelogram, which is $A=ab$. Then the area of a trapezoid is given by the formula:

The area is found as follows:

Another way to express the area of a trapezoid is to say that it is equal to the product of the altitude and the average of the two bases.

The Triangle.

Figure 6A shows a rectangle divided into two equal parts by a diagonal. Each of the two parts of this rectangle is a triangle. A triangle is a plane figure with three straight sides. We know that the area of a rectangle is given by the formula $A=lw$, or $A=ab$. Then the area of a triangle is given by the formula:

Each of the triangles in Figure 6A has a base of 6 yards and an altitude of 3 yards. The area of one triangle is found as follows:

The shaded triangle in Figure 6B has a base of 13 inches and an altitude of 8 inches. The area is $(8)(13) \div 2 = 52$ square inches.

The Circle.

If you will measure the distance around a circular tin can with a tape measure, you will have the circumference (C) of the circle in Figure 7. If you measure the widest distance across the circle (through the center) you will have the diameter (D).

Compare the two measurements and you will find that the circumference is a little more than three times the diameter. This will be true no matter how many circles of different sizes are measured. The ratio is very nearly 3.14, or 3.1416. Given to four decimal places it is 3.1416. For convenience in writing, the number is usually represented by the Greek letter π , called pi. The formula for the circumference of a circle is therefore $C = \pi D$.

One half the diameter is called the radius (r). The radius is the distance from the center to the circumference. In other words, the diameter (D) is 2 times the radius (r). Then, the formula for the circumference of a circle ($C = \pi D$) can be written as follows if we substitute $2r$ for D: $C = 2\pi r$.

The radius of a circular flower bed is 1012 feet. Substituting 3.1416 for π and 1012 for r in the formula, we find the circumference as follows:

Figure 8A shows a circle cut into 16 equal 'pieces of pie.' Figure 8B shows the pieces fitted together to make a figure that looks almost like a parallelogram. The figure would look much more like a parallelogram if the circle were divided into many more pieces. The 'base' of this 'parallelogram' is equal to one half the circumference of the circle. The altitude is equal to the radius.

Since the circumference is equal to $2\pi r$, one half the circumference is equal to πr . Then the area is equal to πr^2 , or πr^2 . This gives us the formula for the area of a circle:

A circular flower bed has a radius of 1012 feet. Substituting 3.1416 (227) for π and 1012 (212) for r, we find the area as follows:

The Measurement of Solids

When we measure the volume of a solid object, such as a box or a brick, we discover how many cubic inches, or cubic feet, or cubic yards it contains.

Rectangular Solid.

If each side, or face, of a solid is a rectangle, the solid is called a **rectangular** solid, or **rectangular** prism. A **rectangular** solid has three dimensions-length, width, and height. If you will look carefully at Figure 9A you will see that it has 6 **rectangular** faces, 12 edges, and 8 corners.

Figure 9B has the same dimensions as Figure 9A. It shows that Figure 9A could contain two layers of 1-inch cubes. The number in each layer is 43, or 12. In the two layers, then, there are 432, or 24 cubic inches. This shows that the formula for finding the volume of a **rectangular** prism is:

By analyzing this formula a little more closely, we also see that the volume of a **rectangular** solid is equal to the area of the base times the height.

If the three dimensions of a **rectangular** prism are all equal, the prism is a cube. Each of the 6 faces of a cube is a square. We find the volume of a cube by cubing one dimension, or edge. Calling the edge e, we have the following formula for the volume of a cube:

We read e^3 as 'e cubed.'

The volume of a 10-inch cube is 103, or 101010, or 1,000 cubic inches.

Using the formula for the volume of a cube, we can rediscover any of the units of cubic measure. The bottom layer of Figure 10 contains 12 rows of 1-inch cubes with 12 cubes in each row. In each layer there are 144 cubes, just as there are 144 square inches in a square foot. In all 12 layers there are 12144 cubic inches, or 1,728 cubic inches.

If we know there are 3 feet in a yard, we can easily rediscover the number of cubic feet in a cubic yard (333, or 33, or 27 cubic feet).

The total surface of a **rectangular** solid consists of the two bases (top and bottom), the two sides, and the two ends. The area of the two bases is $2lw$; the area of the two sides is $2lh$; and the area of the two ends is $2wh$. The total area is given by the formula:

The total area of Figure 9A is $243+242+232$, or $24+16+12$, or 52 square inches.

If an edge of a cube is e , the total surface of a cube is given by the formula:

The Cylinder.

A solid that has the shape of an ordinary tin can or a drum is a cylinder. The two bases of a cylinder are circular in shape.

We have seen that the volume of a **rectangular** solid is equal to the area of the base times the height. The volume of a cylinder is also equal to the area of the base times the height. If the radius of the base is r , the area of the base is r^2 . Then the volume of a cylinder is given by the formula:

Figure 11A shows a can 7 inches in diameter and 6 inches high. Since the diameter is 7 inches, the radius is 3 12 inches. The volume of the cylinder is found as follows:

The surface of a cylinder consists of 2 circular ends and the curved side, which is called the lateral surface. We can find the lateral surface of a tin can by removing the label (Figure 11B). When the label is spread out, it has the shape of a rectangle. The length is the same as the circumference of the circle, or $2r$. The width is the same as the height (h) of the cylinder. Then the area is $2rh$. The area of the 2 ends is $2r^2$. The total area of a cylinder is given by the formula:

The Pyramid.

A pyramid has sloping sides that come to a point at the top. The base may be a triangle, a rectangle, a square, or some other kind of polygon. (A polygon is a plane figure with three or more straight sides.)

A simple experiment shows that the volume of a pyramid is one third the volume of a prism having an equal base and an equal height. Figure 12A shows gravel being poured out of a square pyramid into a square prism. The pyramid and prism have the same base and height. When all the gravel has been poured in, the prism will be one-third full.

The formula for the volume of a **rectangular** prism is $V=lwh$. The formula for the volume of a pyramid is:

in which V is the volume, l is the length of the base, w is the width of the base, and h is the height.

The volume of a pyramid having a 6-inch square base and a height of 8 inches is found as follows: $V = 13668=96$ cubic inches.

The sides of a pyramid are triangles. The total area of the sides is called the convex surface. If the pyramid is a square pyramid, there are four of these triangles. The slant height of the pyramid in Figure 12B is the

altitude of a triangular side (10 inches). Since the area of a triangle is one half the product of the base and the altitude, the total area of all four of these triangles is one half the product of the sum of the bases and the altitude. The sum of the bases in Figure 12B is 16 inches (the perimeter of the base). Then the area of the sides of this pyramid is found as follows: $A = \frac{1}{2} \times 16 \times 10 = 80$ square inches. The total surface area is 80 square inches + 16 square inches (the area of the base), or 96 square inches.

The Cone.

A cone differs from a pyramid in that it has a circular base. The cone represented by Figure 13A has a base with a diameter of 5 inches and a height of 9 inches. The volume of a cone, like the volume of a pyramid, is equal to one third the product of the base area (B) and the height (h); that is:

The area of the base is the area of a circle with a radius of 2.5 inches. Then, the volume of this cone is found as follows, using 3.14 for π : $V = \frac{1}{3} \times 3.14 \times 2.5^2 \times 9 = 58.875$ cubic inches.

The formula for finding the area of the convex surface of a pyramid suggests the method for finding the area of the convex surface of a cone. This area is equal to one half the product of the circumference of the base and the slant height. The circumference of the base in Figure 13B is 3.146, or 18.84 inches. Then the area of the convex surface is $\frac{1}{2} \times 18.84 \times 75.36 = 706.84$, or 706.84 square inches. The area of the base, of course, is $3.14 \times 2.5^2 = 19.625$, or 19.63 square inches. Then the total area of the surface of this cone is 706.84 square inches + 19.63 square inches, or 726.47 square inches.

The Sphere.

Figure 14A shows a cork ball cut in half. A pin was placed in the center of the flat surface of one of the halves and in the center of the curved surface of the other half. Measuring the twine coiled about the pins reveals an interesting fact. The amount required to cover the curved surface is just twice as much as that required to cover the flat surface. This shows that the area of the surface of a hemisphere (a half-sphere) is twice that of a circle having the same radius. We know that the area of a circle is given by the formula: $A = r^2$. Then, the area of the surface of a hemisphere is $2r^2$, and the area of the surface of the entire sphere is given by the formula:

We may think of a sphere as made up of many pieces that resemble pyramids, as shown by Figure 14B. We know that the volume of a pyramid is $V = \frac{1}{3}Bh$. In this case, the height is the same as the radius of the sphere. The volume of each 'pyramid' is therefore $\frac{1}{3}Br$. The sum of the bases of all the pieces is the surface of the sphere, or $4r^2$. Substituting $4r^2$ for B, we have a formula for the volume of the sphere:

$$V = \frac{1}{3} \times 4r^2 \times r, \text{ or } V = \frac{4}{3}r^3$$

If we compare a sphere with a cylinder having a diameter and height equal to the diameter of the sphere, we find the volume of the cylinder is half again as much as the volume of the sphere.

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