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Abstract: Provides information on face-centered cubic packing of spheres as it relates to mathematics, with reference to a study conducted by mathematician Thomas C. Hales of the University of Michigan at Ann Arbor. How the use of this method of packing solved the problem of packing spheres; Comments from Hales; How other scientists view Hales discovery.

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CRACKING KEPLER'S SPHERE-PACKING PROBLEM

The familiar piles of neatly stacked oranges at a supermarket represent a practical solution to the problem of packing spheres as tightly as possible.

Now, a mathematician has proved that no other arrangement of identical spheres fills space more efficiently. That result - if verified - would finally solve a problem that has stymied mathematicians for more than 300 years.

Thomas C. Hales of the University of Michigan in Ann Arbor announced the feat this week and posted his set of proofs on the Internet (http://www.math.lsa.umich.edu/~hales/).

"These results are still preliminary in the sense that they have not been refereed and have not even been submitted for publication," he noted, "but the proofs are - to the best of my knowledge - correct and complete."

The proofs look convincing, says John H. Conway of Princeton University. "Hales has been careful to document everything, so that an auditor who has doubt over any particular point can actually go to the files and check that point."

When supermarket personnel stack oranges, the bottom layer consists of rows that are staggered by half an orange. Placing oranges in the hollows formed by three adjacent oranges in the first layer produces the second layer, and so on. Such an arrangement is known as face-centered cubic packing.

In 1611, Johannes Kepler asserted that this arrangement is the tightest possible way to pack identical spheres. In the 19th century, Carl Friedrich Gauss proved that face-centered cubic packing is the densest arrangement in which the centers of the spheres form a regular lattice. That left open the question of whether an irregular stacking of spheres might be still denser.

In 1953, Lcszl- Fejes T-th reduced the Kepler conjecture to an enormous calculation involving specific cases and later suggested that computers might be helpful for solving the problem. Hales recently worked out a five-step strategy to implement that approach.

In a key step, Michigan graduate student Samuel L.P. Ferguson proved that an irregular arrangement based on a structure known as a pentahedral prism is less dense than the face-centered cubic packing. That was a crucial finding because preliminary computer experiments had indicated that the pentahedral prism might be a counterexample to Kepler's conjecture.

"There were a number of tricky things about solving the problem," Ferguson says. Techniques developed to handle this case were useful in carrying out the other steps, he explains.

The complete, five-part proof appears in a series of articles totaling more than 250 pages. The computer programs and data files take up 3 gigabytes of memory.

Hales used a variety of computational techniques to ensure the accuracy of his calculations. He also worried about the possibility of errors introduced by defective computer chips and any faults in the way a computer translates a program into instructions to a microprocessor.

"There is certainly quite a lot of room for error," Ferguson says. As one check, he and Hales independently wrote computer programs to verify important steps.

Nonetheless, "the problem with such proofs is . . . their length, not the involvement with the computer," Conway says. "A long proof is inevitably weaker than a short one just because there are so many more places where a slip might have been made."

If it holds up, the Hales proof demonstrates that Kepler was right. This feat may not, however, represent the last word on the problem.

"I don't see why there shouldn't be a very short proof involving totally different ideas," Conway says, "and [I] would hazard the guess that there is."

PHOTOS (COLOR): Part of the proof of the Kepler conjecture shows that clusters of identical spheres arranged with face-centered cubic packing (left), like neat stacks of oranges in a grocery store, fill space more efficiently than the pentahedral prism (right).

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By Ivars Peterson
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