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Mean *Median* Surprise

Start with three numbers, say 5, 17, and 23. Their **median** (middle value) is 17. Find a fourth number so that the mean of all four is 17. This number must be 23 ($4 \times 17 - 5 - 17 - 23$).

Repeat the process. The **median** of 5, 17, 23, and 23 is halfway between 17 and 23 (20). Find a fifth number so that the mean of all five numbers is 20. This number is 32 ($5 \times 20 - 5 - 17 - 23 - 23$).

Repeat the process. The **median** of 5, 17, 23, 23, and 32 is 23. Find a sixth number so that the mean of all six is 23. The sixth number must be 38.

Continuing the process, you get the sequence 5, 17, 23, 23, 32, 38, 23, 23, 23, 23, . . . It becomes constant! The same thing happens if you start with the numbers 6, 46, and 78. You get the sequence 6, 46, 78, 54, 66, 74, 96, 108, 102, 110, 96, 100, 195, 213, 96, 96, 96, . . .

It also happens with 13, 41, and 53. You get the sequence 13, 41, 53, 57, 71, 83, 67, 71, 102, 112, 89, 93, 71, 71, 71, . . .

"To our surprise, the same thing happened to every sequence we examined, with whatever three numbers we started," Harris S. Schultz and Ray C. Shiflett report in the May College Mathematics Journal. Schultz is at Cal State Fullerton and Shiflett is at Cal Poly Pomona.

Schultz and Shiflett have dubbed these strings M&m sequences for "mean and **median**."

In these sequences, "we calculate the **median** of the list of the first k values and choose the $k + 1$ value so that the mean of the first $k + 1$ values equals this **median**," the mathematicians note. They offer an Excel spreadsheet for computing M&m sequences at <http://members.cox.net/mathematics/mean-median.xls>.

An M&m sequence is considered stable if it eventually reaches a constant value. The length of the sequence is the number of terms it takes to get to the repeating value for the first time. For example, the sequence starting with 6, 46, and 78 has a stable value of 96 and its length is 15.

The starting numbers don't have to be integers. The numbers 5, 5.5, and 33.9, for example, yield a sequence of length 73 and a stable value -4.65625.

Schultz and Shiflett conjecture that every M&m sequence is stable. In investigating the problem, the mathematicians have so far proved a variety of results that fall short of the ultimate goal but provide useful insights into what's going on.

It's obvious that any sequence that starts with three identical numbers is constant. It's also easy to show that if two of the values are the same, the M&m sequence has length 5. Various other intriguing patterns have also emerged.

Schultz and Shiflett have proved some stability results, particularly for sequences that start with 0, x , and $x + 1$, where x is greater than or equal to 1.

"Our hope is that readers will be motivated to study and explore these M&m sequences," Schultz and Shiflett write.

The question remains: Is every M&m sequence stable?

References:

Shultz, H.S., and R.C. Shiflett. 2005. M&m sequences. *College Mathematics Journal* 36(May):191-198. An Excel spreadsheet for calculating M&m sequences for any set of three integers is available at <http://members.cox.net/mathematics/mean-median.xls>.

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