Building A Box

This lesson uses a real-world situation to help develop students' spatial visualization skills and geometric understanding. Emma, a new employee at a box factory, is supposed to make cube-shaped jewelry boxes. Students help Emma determine how many different nets are possible and then analyze the resulting cubes.

Learning Objectives

Students will:

- Create, compare and describe different two-dimensional nets that can be folded into a three-dimensional cube
- Examine the properties of the nets and resulting cubes, including surface area
- Use rotations and flips to compare various nets

Materials

Computer and Internet connection
Building a Box Activity Sheet
Square Polydron or Geofix pieces, or centimeter grid paper to cut and fold

Instructional Plan

Distribute the Building A Box activity sheet to students, and read the problem aloud to them.


Building A Box Activity Sheet

You may want to review the following terms with students before proceeding with the lesson:

- *cube*
- *edge*
- *square*
- *net*
- *area*
- *surface area*

You might also want to show a small jewelry box to the entire class so that all students understand the situation.

Demonstrate an example of a net that will fold into a cube (such as Figures 1 and 2 on the Building A Box activity sheet). You may also want to give examples of some that do not, such as those shown below.

Nets That Will Not Form Cubes

Discuss why each figure will or will not form a cube, and emphasize that Figures 1 and 2 (on the activity sheet) are two different, non-congruent nets that both fold into cubes. Ask, "Are there other nets that will fold into a cube?" and, "How many different nets are there?"

Provide groups of 3-4 students with centimeter grid paper, square Polydron pieces, or Geofix pieces. (Polydron pieces are available directly from the British company Polydron, and Geofix pieces are available from Didax.) Have them construct, fold and test various nets. The students should record diagrams of all the different nets that they create. You may need to explain that in this activity, nets can only consist of squares.

Students will soon realize that it is always necessary to have exactly six squares in order to form a cube. Ask them to describe other characteristics of the six-square nets that work. Also, have them compare the properties of nets that work with those that do not. In particular, have them address the following questions:

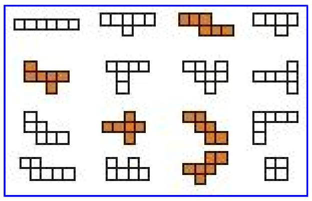
- What are some of the common characteristics of the nets that you created?
- How many squares does each net have?
- How can the squares be arranged? Which arrangements of squares will not form a cube?

Give students time to explore many possibilities, and challenge them to find as many nets as possible. Have students compare seemingly different nets by physically testing for congruency. Ask, "Can one net be moved to fit directly on top of another net?" To help students discover all eleven nets that will form a cube, you may want to encourage an organized approach by asking them to look at how many squares are in the center row. For example, they can build a working net with two, three or four squares in a row, but no acceptable nets have five or six squares in a row. (Note that Figures 1 and 2 on the Building A Box activity sheet both have four squares in a row).

As they explore, have students record properties of their cubes, such as faces, edges, vertices, and surface area, and compare these results to the properties of their nets. They may discover, for instance, that even though the six squares of each net match the six faces of each cube, each net has 14 sides but each cube has only 12 edges.

(In this context, the term *side* indicates a side of one of the squares that lies along the perimeter of the net.) Have students use their 3-D visualization skills to explain this apparent discrepancy; you might need to ask, "What happens to the sides when the nets are folded?"

To conclude the lesson, have groups present their findings the next day and establish classroom solutions. Give students the opportunity to discuss what they have discovered about nets that worked and those that did not, and ask them to compare, generalize, and defend their results. If the class is able to find all eleven nets that are possible, challenge them to explain how they know that those are the only ones that will work. To check their work, students can explore the Cube Nets Tool.

 Cube Nets Tool

The Cube Nets Tool will reveal all eleven nets that form a cube. Therefore, all groups should present their findings to the class before using this tool.

Questions for Students

What properties are common to all nets that will form a cube?

> [All acceptable nets have six squares and 14 sides.]

What type of nets will not work? Why not?

> [Nets with more or fewer than six squares will not work. In addition, many nets with six squares cause two squares to overlap. Obvious cases of this are when four squares share a vertex; when two squares lie on the same side of a center row of squares; and when more than four squares occur in a row.]

Without folding, is there a quick way to determine whether or not a net will fold into a cube?

> [If a net suffers from any of the problems noted above, it will not form a cube, and these problems can be determined by visual inspection.]

How can you determine if two nets are identical?

> [One of the nets will fit exactly on top of another net when flipped or rotated.]

What sort of properties does your final cube have? How do these compare to the properties of the nets?

[The surface area of the cube is equal to the area of the net. The cube has 12 edges, while each net has 14 sides.]

Assessment Options

1. Have students present their results and conclusions to their classmates.

2. Have students write a letter to Emma and Ron explaining and showing their different working nets for a cube. Ask them to describe the properties of a cube and its nets, and require them to explain why they believe they have found all the possible nets.

Extensions

1. Have students determine the net for a typical cereal box. Draw a sketch, and then cut it out and fold it. See if they can design nets for other boxes they have seen. Also, you might have them use the Patch Tool to create nets for other threedimensional objects using triangles, hexagons, and rhombi.

2. Give students the following challenge problem:

    > The ACME box company wants to make these jewelry boxes as efficiently as possible. They can save money by fitting as many nets as possible on one piece of cardboard. If the company use a piece of cardboard that measures 20 cm × 20 cm, how many nets (of any type) can you arrange to fit on one piece of cardboard? You may use any of the working net designs you created and you may arrange them in any way on your piece of cardboard.

    As an alternative, allow students to use the drawing area of the Patch Tool to represent the cardboard, and see how many different nets they can fit into this region.

3. Draw a net on a single sheet of 8½" × 11" piece of paper that will result in the largest cube possible. Which net will you use? What is its volume?

Teacher Reflection

- Which nets were the students able to create right away? Which were more difficult for them to visualize and design? Why?
- What activities could you have done to prepare students for this lesson?
- Did this activity engage the students? What seemed to be the most engaging features of the lesson?
- How did the incorporation of technology influence this lesson and student learning?
- How did this lesson address auditory, tactile, and visual learning styles?
- Did this lesson improve visualization skills? Were students able to determine the connections between 2-D drawings and the 3-D shapes that they can form?
- What content areas did you integrate within the lesson? Was this integration appropriate and successful?
- Did you find it necessary to make adjustments while teaching the lesson? If so, what adjustments, and were these adjustments effective?

NCTM Standards and Expectations

Geometry 3-5

1. Identify and draw a two-dimensional representation of a three-dimensional object.
2. Build and draw geometric objects.
3. Identify and build a three-dimensional object from two-dimensional representations of that object.

Geometry 6-8

1. Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.
2. Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.

References

- Mann, Robert. "Building a Box." *Teaching Children Mathematics*, November 2003, Vol. 10, Num. 3, pp. 150-153.
- NCTM. "Thinking Beyond the Box: Responses to the Building a Box Problem." *Teaching Children Mathematics*, October 2004, Vol. 11, Num. 3, pp. 171-176.

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